Riemann surfaces and algebraic curves, Exercise session 2

(Exercise 1) Desingularise the curve of equation $w^2 + z^5 = 0$ in $\mathbb{C}^2$.

(Exercise 2) Let us consider the *tacnode* quartic D in $\mathbb{C}^2$ with equation $w(w - z^2) - z^4 = 0$. Show that D is desingularised after two blow-ups. (First blow-up in a neighbourhood U of $p = (0, 0)$ at p. Denote with $\pi : \tilde{U} \to U$ such blow-up. The strict transform C' of C via π has a nodal point at $q = ((0, 0), [1, 0]) \in \tilde{U}$. Then it is sufficient to blow-up $\tilde{U}$ at q.)

(Exercise 3) Desingularise the plane curve $C = \{[x : y : z] : z^{2g}y^2 - \prod_{i=1}^{2g+2}(x - a_i z) = 0 : a_i \neq a_j \forall i \neq j\}$ (see Exercise (3) of Exercise session 1).

(Exercise 4) Let us consider the same objects of Exercise (1) in Exercise sheet 1. Study $f = \frac{x}{y}$ in $C \cap \{y = 0\}$. Choose a local chart. Describe f in such chart. Does f have poles?

(Exercise 5) Let us consider the same objects of Exercise (2) in Exercise sheet 1. Study $f = \frac{x}{y}$ in $C \cap \{y = 0\}$. Choose a local chart. Describe f in such chart. Does f have poles?