Riemann surfaces and algebraic curves, Exercise session 3

(Exercise 1) Let us consider the curve

$$\Gamma = \{[x : y : z] \in \mathbb{P}^2_{\mathbb{C}} | xyz^3 + x^5 + y^5 = 0\}.$$

Let $\sigma : C \to \Gamma$ be the desingularisation of Γ. Let $p = [0 : 0 : 1]$.

- Show that $\sigma^{-1}(p)$ consists of two points $\sigma^{-1}(p) = \{p_1, p_2\}$.
- Let $f = x \circ \sigma$ and $h = y \circ \sigma$. Describe zeros, poles, ramification order, ect... of f, h and f/h.

(Exercise 2) Show that every complex torus $\mathbb{C}/L$ is isomorphic to a torus which has the form $\mathbb{C}(\mathbb{Z} + \tau\mathbb{Z})$, where τ is a complex number with strictly positive imaginary part.