Riemann surfaces and algebraic curves, Exercise session 4

Let Λ and Λ' be lattices $\{m\tau_1 + n\tau_2 : m, n \in \mathbb{Z}\}$ and $\{m\tau_1' + n\tau_2' : m, n \in \mathbb{Z}\}$.

(Exercise 1) Let $\phi : \mathbb{C}/\Lambda \to \mathbb{C}/\Lambda'$ sending $z + \Lambda$ to $mz + b + \Lambda'$. Show that ϕ is a group homorphism iff $b \in \Lambda'$ iff $\phi(0) = 0$.

(Exercise 2) Let $\phi : \mathbb{C}/\Lambda \to \mathbb{C}/\Lambda$ sending $z + \Lambda$ to $-z + \Lambda$.

- Show that ϕ preserves the Weierstrass embedding of the complex torus in an elliptic curve E in $\mathbb{P}^2_{\mathbb{C}}$.
- Show that ϕ corresponds $\mathbb{P}^2_{\mathbb{C}}$ to $\psi : E \to E$ sending $[x : y : z]$ to $[x : -y : z]$.
- Show that the 2-torsion elements in $\mathbb{C}/\Lambda$ correspond to the points $[x : 0 : z]$ in E.