Riemann surfaces and algebraic curves, Exercise session 6

From **Exercise sheet 5**:
Let $\Lambda = \mathbb{Z}\tau_1 + \mathbb{Z}\tau_2$ be a lattice in $\mathbb{C}$ and let $\wp$ be the Weierstrass $\wp$-function for the lattice Λ. Let $g_2, g_3 \in \mathbb{C}$ such that $(\wp')^2 = 4\wp^3 - g_2\wp - g_3$. The subset

$$C = \{[x : y : h : t] \in \mathbb{P}^3_{\mathbb{C}} : y^2 = 4xt - g_2xh - g_3h^2, x^2 = ht\},$$

is a submanifold of $\mathbb{P}^3_{\mathbb{C}}$.
The map

$$\mathbb{C}/\Lambda := T \to \mathbb{P}^3_{\mathbb{C}},$$

sending $z \mapsto [\wp(z) : \wp'(z) : 1 : \wp^2(z)]$ for $z \neq 0$ and $0 \mapsto [0 : 0 : 0 : 1]$ is a holomorphic map.

(Exercise 1) Prove that $\phi(T) = C$.

(Exercise 2) The map ϕ is injective and has maximal rank 1 on T.