UNIVERSITÄT TÜBINGEN
FACHBEREICH MATHEMATIK

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Exercises Geometric Group Theory

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Exercise sheet 11

Hand in exercises 2(1), 3(a) by July 13, 11:59 a.m. The remaining exercises will be carried out during the exercise session.
Group 2 (respectively Group 1) can deliver the exercise sheets to Manzaroli mailbox in room A16 building C on the 3rd floor or scanned and sent via email to Manzaroli Matilde (respectively to Loujean Cobigo).

Exercise 1

Let $a, a' \in \mathbb{R}_{>1}$ and $f : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}, x \mapsto a^x$ und $g : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}, x \mapsto a'^x$ generalised growth functions. Show that f and g are quasi-equivalent.

Exercise 2

Let $H := \langle x, y, z | z = xyx^{-1}y^{-1}, xz = zx, yz = zy \rangle$ be the discrete Heisenberg group.

1. Let l be a positive integer. Let us consider a ball B of radius l and center the identity in the Cayley graph of the Heisenberg group. For $n = 0, 1, 2$ draw and count the elements in B such that the exponent of x sum up to n.
2. Show that H grows polynomially of degree 4.

(*Hint: Consider the Cayley graph of H in $\mathbb{Z}^3$.*)

Exercise 3

Let G be a group.

(a) Show for all $j \in \mathbb{N}$ that $C_{j+1}(G) \subset C_j(G)$ is a normal subgroup and the quotient ${}^{C_j(G)}/_{C_{j+1}(G)}$ is abelian.

(b) For all $j \in \mathbb{N}$ conclude that $G^{(j+1)} \subset G^{(j)}$ is a normal subgroup and the quotient ${}^{G^{(j)}}/_{G^{(j+1)}}$ is abelian.
