UNIVERSITÄT TÜBINGEN
FACHBEREICH MATHEMATIK

Loujean Cobigo
Matilde Manzaroli
Hannah Markwig

Exercises Geometric Group Theory

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Exercise sheet 7

Hand in exercises 1a), 1b), 3 by June 15, 11:59 a.m. The remaining exercises will be carried out during the exercise session.
Group 2 (respectively Group 1) can deliver the exercise sheets to Manzaroli mailbox in room A16 building C on the 3rd floor or scanned and sent via email to Manzaroli Matilde (respectively to Loujean Cobigo).

Exercise 1

In this exercise we want to treat $\mathbb{Z}$ as a finitely generated group.

(a) Show that a quasi-isometric embedding $f : \mathbb{Z} \to \mathbb{Z}$ is already a quasi-isometry.

(b) Let $X := (\mathbb{Z} \times \{0\}) \cup (\{0\} \times \mathbb{Z}) \subset \mathbb{Z}^2$ be a metric space equipped with the Euclidean metric of $\mathbb{R}^2$. Show there exists no quasi-isometric embedding $f : X \to \mathbb{Z}$.

(c) Decide whether $\mathbb{Z}$ and $\mathbb{Z}^2$ are quasi-isometric and prove your claim.

(d) Show that $\mathbb{Z}$ and $\mathbb{F}_2$ with two generators are not quasi-isometric.

Exercise 2

Let X, Y be be finitely generated groups and $f : X \to Y$ a bijective quasi-isometry. Show that f is a bilipschitz equivalence.

Exercise 3

Let G be a finitely generated group and N a normal subgroup of G. Show that N is finite if and only if the projection $\pi : G \to {}^{G}/_{N}$ is a quasi-isometry.

(*Hint: For the reverse direction, show that N is contained in a suitable sphere of finite radius.*)
