UNIVERSITÄT TÜBINGEN
FACHBEREICH MATHEMATIK

Loujean Cobigo
Matilde Manzaroli
Hannah Markwig

Exercises Geometric Group Theory

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Exercise sheet 8

Hand in exercises 1(a), 1(b), 3 by June 22, 11:59 a.m. The remaining exercises will be carried out during the exercise session.
Group 2 (respectively Group 1) can deliver the exercise sheets to Manzaroli mailbox in room A16 building C on the 3rd floor or scanned and sent via email to Manzaroli Matilde (respectively to Loujean Cobigo).

Exercise 1

(a) Show that $\mathbb{R}^2\setminus\{0\}$ equipped with the Euclidean metric is not geodesic.

(b) Justify the fact that $\mathbb{R}^2\setminus\{0\}$ is $(1,\epsilon)$-quasi-geodesic for all $\epsilon > 0$.

(c) Decide whether the following statement is true or not and prove your claim:
If X is geodesic and $f : X \to Y$ a quasi-isometry, then Y is also geodesic.

Exercise 2

Let $X := \{(a^3, b)|a, b \in \mathbb{Z}\}$ a metric spece with the standard metric of $\mathbb{Z}^2$. Then $\mathbb{Z}$ operates on X as $\big(z, (a^3, b)\big) \mapsto (a^3, b + z)$. Check whether you can apply the Švarc-Milnor Lemma.

Exercise 3

Let (X, d) a metric space on which a group G operates via isometries such that for all $x \in X$ and $r \geq 0$ the set $\{g \in G|d(x, g\cdot x) \leq r\}$ is finite. Show that for every closed ball $B \subset X$ the set $\{g \in G|g\cdot B \cap B \neq \emptyset\}$ is finite.

Exercise 4

Let (X, d) a proper metric space on which a group G operates via isometries such that every orbite of this operation is quasi-dense in X. Show that this operation is cocompact.

(*Hint: Recall that images of compact sets are compact under continuous maps.*)
